Work & Energy

Chapters 7 & 8

KINETIC ENERGY AND WORK

Energy

- Energy is defined as a measure of the ability to do scientific work.
- It is a *scalar quantity* associated with the state (or condition) of one or more objects.
- The property of an object that enables it to do work.
- Measured in joules (J) $J = \frac{kg \cdot m^2}{s^2} = N \cdot m$

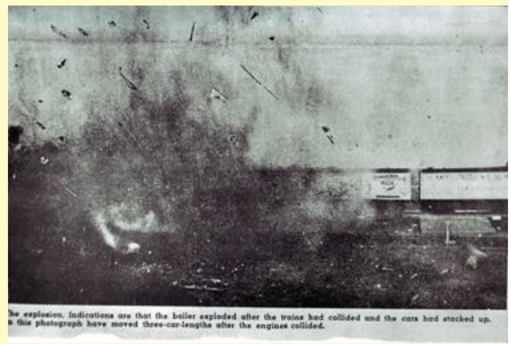
Kinetic Energy (K)

- Energy due to an object's motion.
- Energy associated with the state of motion of an object.
- Energy of motion.
- The faster an object moves, the more kinetic energy it has.
- Units: joules (J)

$$\boxed{K = \frac{1}{2}mv^2}$$

Sample 1

In 1896 in Waco, Texas, William Crush parked two locomotives at opposite ends of a 6.4 km long track, fired them up, tied their throttles open, and then allowed them to crash head-on at full speed in front of 30,000 spectators. Hundreds of people were hurt by flying debris; several were killed. Assuming each locomotive weighed 1.2×10^6 N and its acceleration was a constant $0.26\ m/s^2$, what was the total kinetic energy of the two locomotives just before the collision?



Work

- When you change the kinetic energy of an object you are transferring energy either to or from the object. In such a transfer of energy via a force, work (W) is said to be done on the object by the force.
- Work is energy transferred to or from an object by means of a force acting on the object.
- Energy transferred to the object is positive work, and energy transferred from the object is negative work.
- "Work" is transferred energy; "doing work" is the act of transferring the energy.
- Work is a scalar quantity and is measured in joules

Work

To calculate the work a forces does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\phi$$

Caution: The above equations have two restrictions:

(1) The force must be a constant force, i.e. it must not change in magnitude or direction as the object moves

(2) The object must be particle like, or rigid, with all parts moving together in the same direction

Signs for Work

- A forces does positive work when it has a vector component in the same direction as the displacement ($\phi < 90°$)
- It does negative work when it has a vector component in the opposite direction ($\phi > 90°$)
- It does zero work when it has no such vector component ($\phi = 90°$)

Net Work

When two or more force act on an object, the net work done on an object is the sum of the works done by the individual forces. We can calculate the net work in two ways:

1. We can find the work done by each force and then sum those works
2. We can find the net force of all the forces and use that vector in the work equation

Work-Kinetic Energy Theorem

$$\boxed{\Delta K = K_f - K_i = W}$$

- The change in kinetic energy of an object is equal to the net work done on that object.
- Theorem that states that whenever work is done, energy changes.
- If the net work done on a particle is positive, then the particle's kinetic energy increases by the amount of the work. If the net work done is negative, then the particle's kinetic energy decreases by the amount of the work.

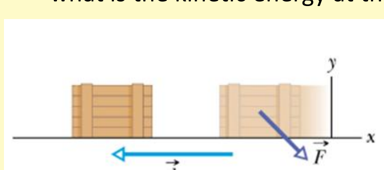
Checkpoint 1

A particle moves along an *x* axis. Does the kinetic energy of the particle increase, decrease, or remain the same if the particle's velocity changes (a) from -3 m/s to -2 m/s and (b) from – 2 m/s to 2 m/s? (c) In each situation, is the work done on the particle positive, negative, or zero?

Sample 2

During a storm, a crate of crepe is sliding across a slick oily parking lot through a displacement ***d*** = (-3.0 m)î while a steady wind pushes against the crate with a force ***F*** = (2.0 N)î + (-6.0 N)ĵ. The situation and coordinate axes are shown below.

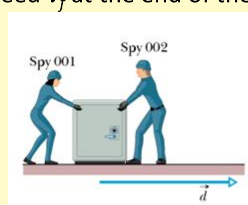
(a) How much work does this force do on the crate during the displacement?
(b) If the crate has a kinetic energy of 10 J at the beginning of displacement ***d***, what is the kinetic energy at the end of ***d***?



Sample 3

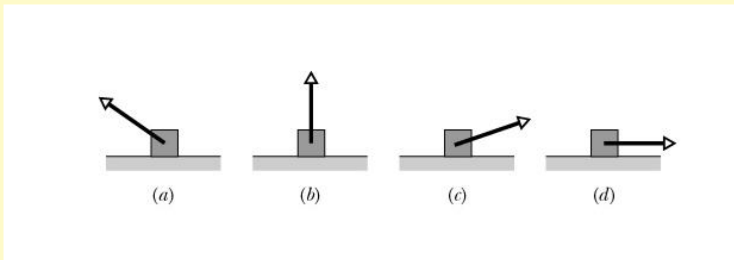
The figure below shows two industrial spies sliding an initially stationary 225 kg floor safe a a displacement ***d*** of magnitude 8.50 m, straight toward their truck. The push $\boldsymbol{F_1}$ of spy 001 is 12.0 N, directed at an angle of 30.0° downward from the horizontal; the pull $\boldsymbol{F_2}$ of spy 002 is 10.0 N, directed at 40.0° above the horizontal. The magnitudes and directions of these forces do not change as the safe moves, and the floor and safe make frictionless contact.

(a) What is the net work done on the safe by forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ during the displacement ***d***?
(b) During the displacement, what is the work W_g done on the safe by the gravitational force $\boldsymbol{F_g}$ and what is the work W_N done on the safe by the normal force $\boldsymbol{F_N}$ from the floor?
(c) The safe is initially stationary. What is its speed v_f at the end of the 8.50 m displacement?



Checkpoint 2

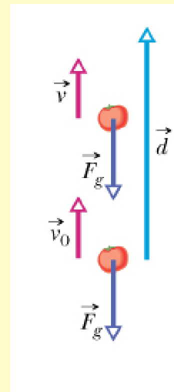
The figure shows four situations in which a force acts on a box while the box slides rightward a distance *d* across a frictionless floor. The magnitudes of the forces are identical; their orientations are as shown. Rank the situations according to the work done on the box during the displacement, most positive first.



WORK DONE BY PARTICULAR FORCES

Work Done by the Gravitational Force

Suppose you throw a tomato into the air, it will change velocity continuously during its flight. We can now associate this changing velocity with a changing kinetic energy. Because the kinetic energy of the tomato changes, the gravitational force must be doing work on the tomato.

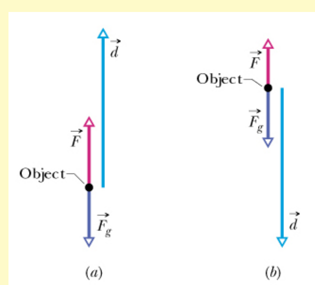


$$W_g = mgd\cos\phi$$

The work done by gravity is negative as the tomato rises (ϕ = 180°) and positive as the tomato falls (ϕ = 0°)

Work Done in Lifting and Lowering and Object

When analyzing a situation in which an object is being lifted or lowered vertically, it is important to remember that the gravitational force still does work as well as the applied force.



$$\Delta K = K_f - K_i = W_{app} + W_g$$

If the object is *stationary before and after* the movement, the change in kinetic energy is zero and the work done by the applied force is equal but opposite to the work done by gravity.

$$W_{app} = -W_g$$

Sample 4

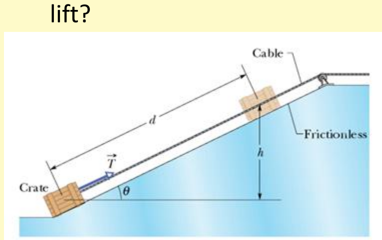
In the 1950's, Paul Anderson became legendary for lifting tremendous loads. One of his lifts remains a record: Anderson stooped beneath a reinforced wood platform, placed his hands on a short stool to brace himself, and then pushed upwards on the platform with his back, lifting the platform straight up a distance of about 1.0 cm. The platform held automobile parts and a safe filled with lead, with a total what of 27,900 N (6270 lb).

(a) As Anderson lifted the load, how much work was done on it by the gravitational force $\boldsymbol{F_g}$?
(b) How much work was done by the force Anderson applied to make the lift?

Sample 5

An initially stationary 15.0 kg crate of cheese wheels is pulled, via a cable, a distance d = 5.70 m up a frictionless ramp to a height h of 2.50 m, where it stops.

(a) How much work W_g is done on the crate by the gravitational force $\boldsymbol{F_g}$ during the lift?
(b) How much work W_T is done on the crate by $\boldsymbol{T}$ from the cable during the lift?



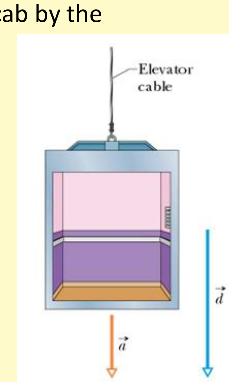
Checkpoint 3

Suppose we raise the crate in sample 5 by the same height h but with a longer ramp. (a) Is the work done by force $\boldsymbol{T}$ now greater than, smaller than, or the same as before? (b) Is the magnitude of $\boldsymbol{T}$ need to move the crate now greater than, smaller than, or the same as before?

Sample 6

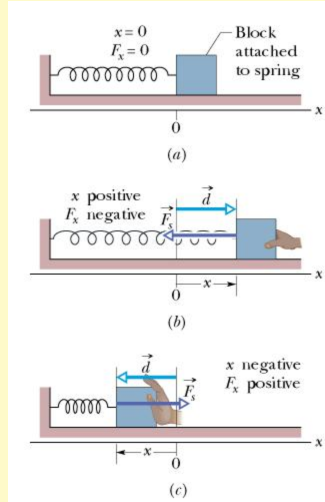
An elevator cab of mass m = 500 kg is descending with speed v_i = 4.0 m/s when its supporting cable begins to slip, allowing it to fall with constant acceleration $\boldsymbol{a} = \boldsymbol{g}/5$.

(a) During the fall through a distance d = 12 m, what is the work W_g done on the cab by the gravitational force $\boldsymbol{F_g}$?
(b) During the 12 m fall, what is the work W_T done on the cab by the gravitational force $\boldsymbol{F_g}$?



The Spring Force

- Figure (*a*) shows a spring in its *relaxed state*
  - It is neither stretched nor compressed
- In Figure (*b*) the spring is stretched from its relaxed position by a distance *x*
- In Figure (*c*) the spring is compressed from its relaxed position by a distance *x*
- In both Figures (*b*) and (*c*), if released the spring will contract back to its relaxed state, the force that causes this motion is known as the spring force
  - The spring force is sometimes said to be a *restoring force* because it restores the springs relaxed position.
- The spring force is a variable (non-constant) force
  - The further it is stretched or compressed, the larger the force



Hooke's Law

- The spring force is given by Hooke's Law:

$$F_s = -kx$$

- The minus sign indicates that the direction of the spring force is always opposite the direction of the displacement
- The constant *k* is called the spring constant and is a measure of stiffness of the spring
  - The larger *k* the stiffer the spring
  - Units: N/m
- Note that the spring force is a function of position: $F_s(x)$
- Hooke's law shows a linear relationship between *F* and *x*

The Work Done by a Spring Force

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

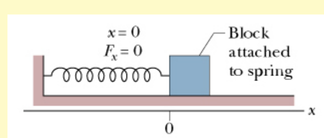
- Work W_s is positive if the block ends up closer to the relaxed position (x = 0) than it was initially. It is negative if the block ends up farther away from x = 0. It is zero if the block ends up at the same distance from x = 0.

- Just like with the gravitational force, if a block is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

$$\Delta K = K_f - K_i = W_{app} + W_s \qquad W_{app} = -W_s$$

Checkpoint 4

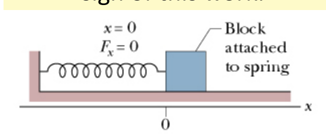
For three situations, the initial and final positions, respectively , along the x axis for a block attached to a spring are (a) -3 cm, 2cm; (b) 2 cm, 3cm; and (c) -2 cm, 2 cm. In each situation, is the work done by the spring force on the block positive, negative, or zero?



Sample #7

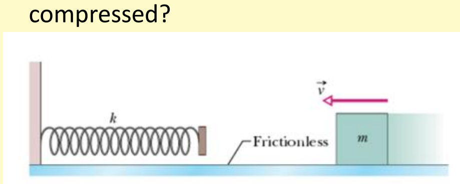
A package of spicy Cajun pralines lies on a frictionless floor, attached to the free end of a spring as shown below. A rightward applied force of magnitude F_a = 4.9 N would be needed to hold the package at x_1 =12 mm.

a) How much work does the spring force do on the package if the package is pulled rightward from x_0 = 0 to x_2 = 17 mm?
b) Next, the package is moved leftward to x_3 = -12 mm. How much work does the spring force do on the package during this displacement? Explain the sign of this work.



Sample Problem #8

In the figure below, a cumin canister of mass m = 0.40 kg slides across a horizontal frictionless counter with a speed v = 0.50 m/s. It then runs into and compresses a spring of spring constant k = 750 N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed?

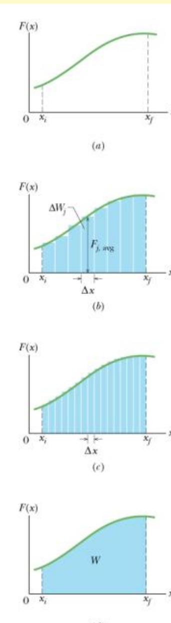


WORK DONE BY A GENERAL VARIABLE FORCE

One-Dimensional Analysis of a Variable Force

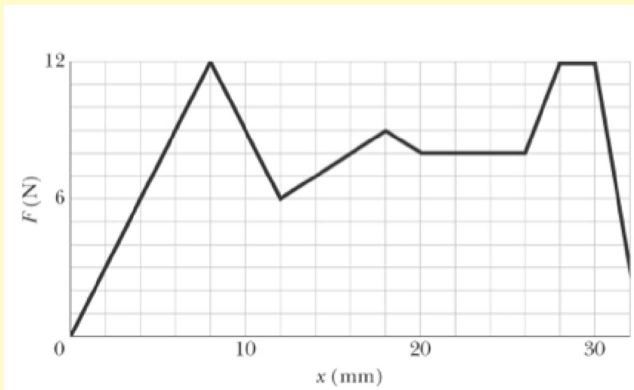
- Suppose you want to determine the work done by a force that varies, nonlinearly, with position, such as the one represented by the graph in Figure (*a*).
- We cannot use our basic force equation here because *F* is not constant
- However, if we divided the graph into smaller and smaller segments we can find fairly constant values of *F*. (Figures (*b)* and (*c*)]
- This is the same as taking the integral of the function *F(x)*

$$W = \int_{x_i}^{x_f} F(x)dx$$



Sample Problem #9

In an epidural procedure, the feel a doctor has for the needles penetration is the variable force that must be applied to advance the needle through the tissues. The figure below is a graph of the force magnitude *F* versus displacement *x* of the needle tip for a typical epidural procedure. (The line segments have been straightened somewhat from the original data) How much work is done by the force exerted on the needle to get the needle to the epidural space at 30 mm?



Sample 10

Force $\vec{F} = (3x^2 N)\hat{i} + (4N)\hat{j}$, with *x* in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2 m, 3 m) to (3 m, 0 m)? Does the speed of the particle increase, decrease, or remain the same?

POWER

Power

- Power is defined as the time rate at which work is done by a force.

$$P_{avg} = \frac{W}{\Delta t} \qquad P = \frac{dW}{dt}$$

- Unit: J/s = W (watts)
- Power can also be expressed in terms of the force on a particle (***F***) and the velocity of the particle (***v***)

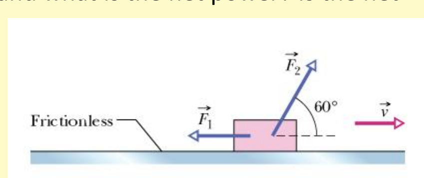
$$P = \vec{F} \cdot \vec{v}$$

Checkpoint 5

A block moves with uniform circular motion because a cord tied to the block is anchored at the center of the circle. Is the power due to the force on the block from the cord positive, negative, or zero?

Sample 11

The figure shows constant force $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ acting on a box as the box slides rightward across a frictionless floor. Force $\boldsymbol{F_1}$ is horizontal, with magnitude 2.0 N; force $\boldsymbol{F_2}$ is angled upward by 60° to the floor and has magnitude 4.0 N. The speed v of the box at a certain instant is 3.0 m/s. What is the power due to each force acting on the box at that instant, and what is the net power? Is the net power changing at that instant?



POTENTIAL ENERGY

Potential Energy (U)

- Energy due to an object’s position.
- Energy of position, usually related to the relative position of two things.
- Energy stored by an object by virtue of it’s position.
- Units: joules (J)

Types of Potential Energy

Gravitational Potential Energy (GPE)

- Potential energy due to elevated positions.
- Energy associated with the state of separation between two objects that attract each other by the gravitational force.
- Units: joules (J)

Elastic Potential Energy (EPE)

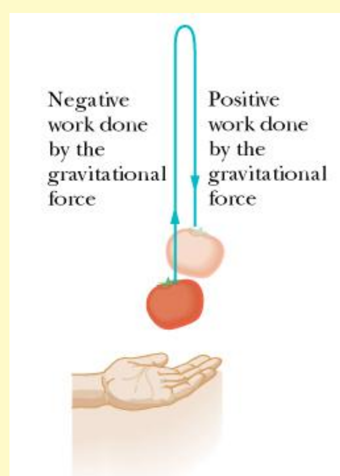
- Energy associated with the state of compression or extension of an elastic object.
- Units: joules (J)

Work & Potential Energy

- If you throw a an object in the air, for either the rise or the fall, the change ΔU in gravitational potential energy is defined as being equal to the negative of the work done on the tomato by the gravitational force.

$$\Delta U = -W$$

- This equation also applies to a block-spring system
- This is because both the gravitational force and the spring force are conservative forces.



Explanation of Conservative Forces

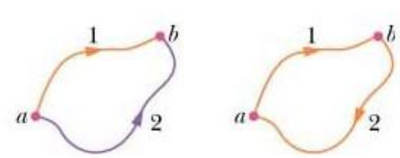
The key elements that the gravitational force and the spring force have in common:

- The system consists of two or more objects
- A force acts between a particle-like object in the system and the rest of the system.
- When the system configuration changes, the forces does work (W_1) on the object, transferring energy between the kinetic energy of the object and some other type of energy of the system.
- When the configuration change is reversed, the forces reverses the energy transfer, doing work W_2 in the process.

- In a situation in which $W_1 = -W_2$ is always true the other type of energy is potential energy and the force is said to be conservative.

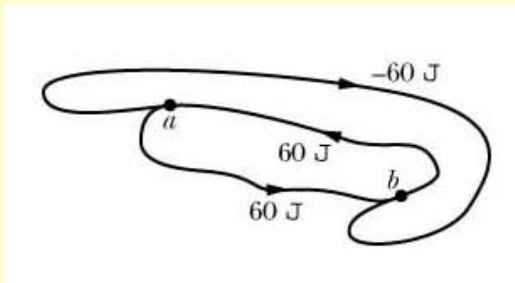
Conservative vs. Nonconservative Forces

- Conservative Forces
  - The net work done by a conservative force on a particle moving around a closed path is zero
  - The work done by a conservative force on a particle moving between two points does not depend on the path taken by the particle (path independent)
  - Examples: gravitational force and spring force
- Nonconservative Forces
  - Net work around a closed path is non-zero
  - Path dependent
  - Examples: friction and drag



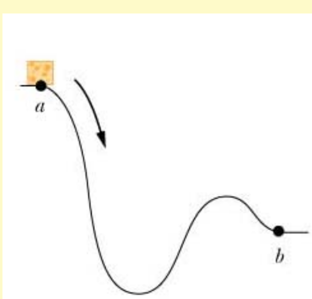
Checkpoint 6

The figure shows three paths connecting points *a* and *b*. A single force ***F*** does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force ***F*** conservative?



Sample 12

In the figure below a 2.0 kg block of slippery cheese that slides along frictionless track from point *a* to point *b*. The cheese travels through a total distance of 2.0 m along the track, and a net vertical distance of 0.80 m. How much work is done on the cheese by the gravitational force during the slide?



Potential Energy

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

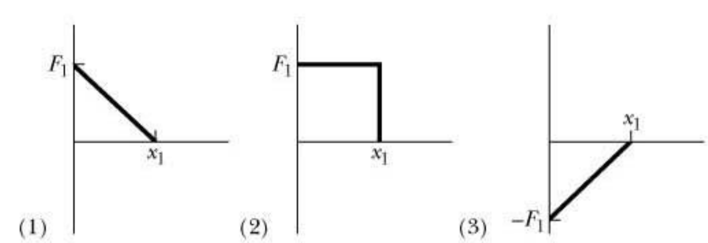
$$U_g(y) = mgy$$

$$U_s(x) = \frac{1}{2}kx^2$$

Note: The gravitational potential energy associated with a particle-Earth system depends only on the vertical position *y* relative to the reference position

Checkpoint 7

A particle is to move along an *x* axis from $x = 0$ to x_1 while a conservative force, directed along the *x* axis, acts on the particle. The figure shows three situations in which the *x* component of that forces varies with *x*. The force has the same maximum magnitude F_1 in all three situations. Rank the situations according to the change in the associated potential energy during the particle's motion, most positive first.

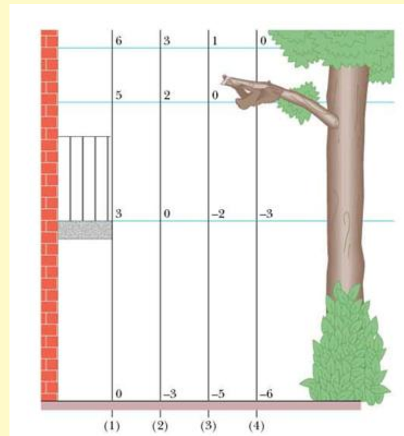


Sample 13

A 2.0 kg sloth hangs 5.0 m above the ground.

a) What is the gravitational potential energy *U* of the sloth-Earth system if we take the reference point y = 0 to be (1) at the ground, (2) at a balcony floor that is 3.0m above the ground, (3) at the limb, and (4) 1.0 m above the limb? Take the gravitational potential energy to be zero at y = 0.

b) The sloth drops to the ground. For each choice of reference point, what is the change Δ*U* in the potential energy of the sloth-Earth system due to the fall?



Problem-Solving Tactic

A potential energy is associated with a system as a whole. However, you might see statements that associate it only with one part of the system. For example, you might read, "An apple hanging in a tree has a gravitational potential energy of 30 J." Such statements are often acceptable, but you should always keep in mind that the potential energy is actually associated with a system – here the apple-Earth system. Also keep in mind that assigning a particular potential energy value, such as 30 J here, to an object or even a system makes sense *only* if the reference potential energy value is known.

CONSERVATION OF MECHANICAL ENERGY

Mechanical Energy (E_{mec})

- The sum of potential energy and kinetic energy.

$$E_{mec} = K + U$$

- The energy of an object due to its position and motion.

- Units: joule (J)

The Principle of Conservation of Mechanical Energy

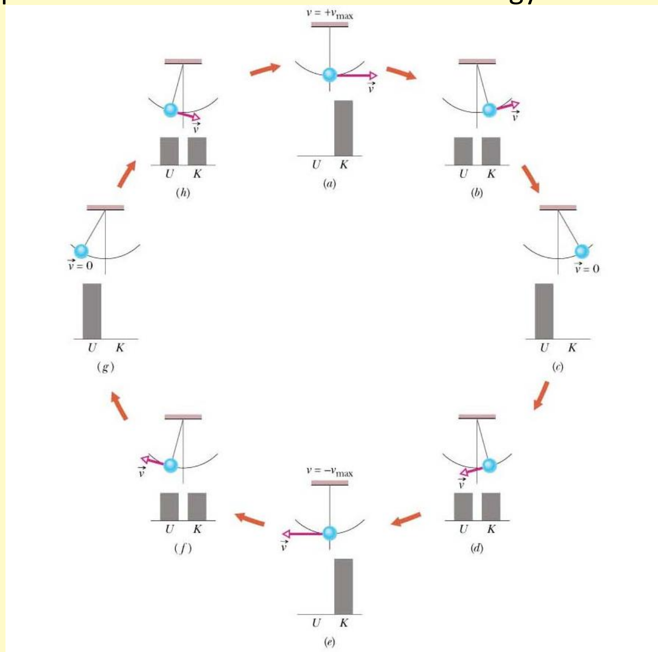
- In an isolated system where only conservative forces cause energy changes, the kinetic energy and the potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.
  - i.e. Mechanical energy is conserved if only conservative forces act

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

- When the mechanical energy of a system is conserved, we can relate the mechanical energy at one instant to that at another instant without considering the intermediate motion and without finding the work done by the forces involved

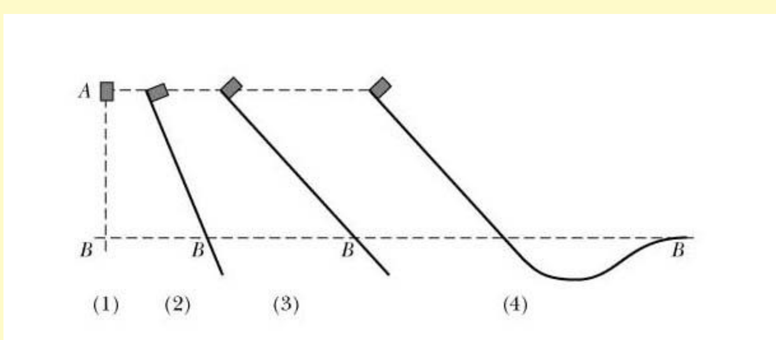
$$K_2 + U_2 = K_1 + U_1$$

Example of Conservation of Mechanical Energy – Ideal Pendulum



Checkpoint 8

The figure shows four situations: one in which an initially stationary block is dropped and three in which the block is allowed to slide down frictionless ramps. (a) Rank the situations according to the kinetic energy of the block at point *B*, greatest first. (b) Rank them according to the speed of the block at point *B*, greatest first.



Sample 14

A child of mass *m* is released from rest at the top of a water slide, at height $h = 8.5$ m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.

Problem-Solving Tactics

Asking the following questions will help you to solve problems involving the principle of conservation of mechanical energy.

- For what system is mechanical energy conserved?
  - You should be able to separate your system from its environment.
- Is friction or drag present?
  - If yes, mechanical energy is NOT conserved because these are nonconservative forces
- Is your system isolated?
  - Conservation of energy only applies to isolated systems. That means that no external forces should do work on objects in the system.
- What are the initial and final states of your system?
  - The system changes from some initial configuration to some final configuration. You apply the principle of conservation of mechanical energy by saying that E_{mec} has the same value in both configurations. Be very clear about what these two configurations are.

POTENTIAL ENERGY FUNCTIONS

Finding Force from Potential Energy

- We already know that we can find the change in potential energy between two points if we know the force function:

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

- Hopefully you can see that we can also go the other way and find force if we know the function for the potential energy:

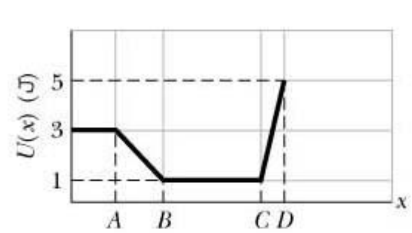
$$\boxed{F(x) = -\frac{dU(x)}{dx}}$$

Reading Potential Energy Curves

- Turning Point
  - $K = 0$ and the particle changes direction.
- Neutral Equilibrium
  - $K = 0$ and the particle is stationary
    - Ex: Marble placed on horizontal tabletop
- Unstable Equilibrium
  - $K = 0$, the particle is stationary, however it is even slightly displaced in either direction, a nonzero force pushes it further in the same direction and the particle continues to move
    - Ex: Marble placed on top of a bowling ball
- Stable Equilibrium
  - $K = 0$, the particle is stationary, and if displaced slightly left or right a restoring force appears to move it back to its stationary position
    - Ex: Marble placed in a bowl.

Checkpoint 9

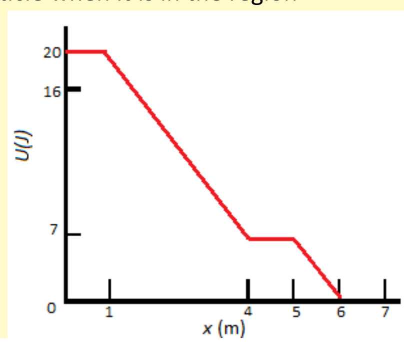
The figure gives the potential energy function *U(x)* for a system in which a particle is in one-dimensional motion. (a) Rank regions *AB*, *BC*, and *CD* according to the magnitude of the force on the particle, greatest first. (b) What is the direction of the force when the particle is in region *AB*?



Sample 15

A 2.00 kg particle moves along an x axis in one-dimensional motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted below. That is, if the particle were placed at any position between $x = 0$ and $x = 7.00$ m, it would have the plotted value of U. At $x = 6.5$ m, the particle has a velocity $v_0 = (-4.00\text{ m/s})\hat{\imath}$.

(a) From the plot, determine the particle's speed at $x_1 = 4.5$ m.
(b) Where is the particle's turning point located?
(c) Evaluate the force acting on the particle when it is in the region 1.9 m < x < 4.0 m.



Work Done on a System by an External Force

- Work is energy transferred to or from a system by means of an external force acting on the system

$$W = \Delta K + \Delta U$$

$$W = \Delta E_{mec}$$

WORK DONE BY EXTERNAL FORCES

Work Done on a System by an External Force

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$$W = \Delta K + \Delta U$$

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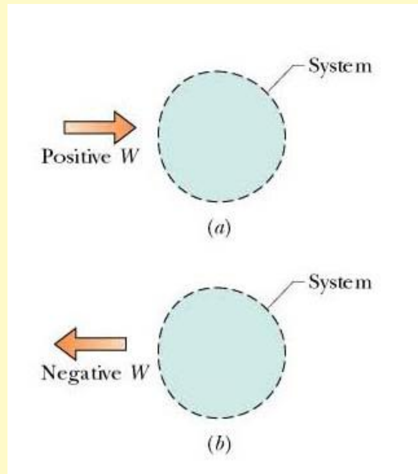
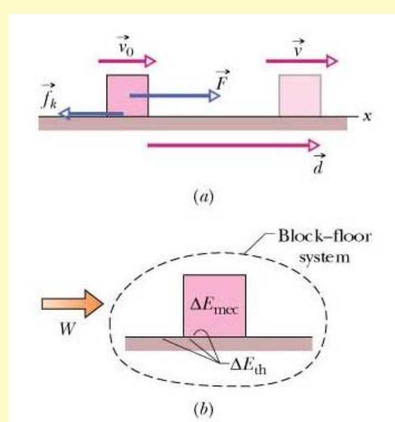


Work Done by Friction

When a moving object experiences friction, some of its mechanical energy is converted into thermal energy, E_{th}.

$$\Delta E_{th} = f_k d$$

$$W = \Delta E_{mec} + \Delta E_{th}$$



Checkpoint 10

In three trials, a block is pushed by a horizontal applied force across a floor that is not frictionless. The magnitude *F* of the applied force and the results of the pushing on the block's speed are given in the table. In all three trials, the block is pushed through the same distance *d*. Rank the three trials according to the change in thermal energy of the block and floor that occurs in that distance *d*, greatest first.

Trial	F	Result on Block's Speed
a	5.0 N	decreases
b	7.0 N	remains constant
c	8.0 N	Increases

Sample 16

The prehistoric people of Easter Island carved hundreds of gigantic stone statues in a quarry and then moved them to sites all over the island. How they managed to move the statues by as much as 10 km without the use of sophisticated machines has been hotly debated. They most likely cradled each statue in a wooden sled and then pulled the sled over a "runway" consisting of almost identical logs acting as rollers. In a modern reenactment of this technique, 25 men were able to move a 9000 kg Easter Island-type statue over 45 m over level ground in 2 minutes.

(a) Estimate the work the net force ***F*** from the men did during the 45 m displacement of the statue, and determine the system on which that force did work
(b) What was the increase ΔE_{th} in the thermal energy of the system during the 45 m displacement?
(c) Estimate the work that would have been done by the 25 men if they had moved the statue 10 km across level ground on Easter Island. Also estimate the total change ΔE_{th} that would have occurred in the statue-sled-logs-ground system

Sample 17

A food shipper pushes a wood crate of cabbage heads (total mass m = 14 kg) across a concrete floor with a constant horizontal force $\boldsymbol{F}$ of magnitude 40 N. In a straight-line displacement of magnitude d = 0.50 m, the speed of the crate decreases from v_0 = 0.60 m/s to v = 0.20 m/s.

(a) How much work is done by force $\boldsymbol{F}$, and on what system does it do the work?
(b) What is the increase ΔE_{th} in the thermal energy of the crate and floor?

CONSERVATION OF ENERGY

The Law of Conservation of Energy

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{\text{int}}$$

- The total energy of a system can change only by amounts of energy that are transferred to or from the system.
- The total energy of an isolated system cannot change.
- In an isolated system, we can relate the total energy at one instant to the total energy at another instant without considering the energies at intermediate times.

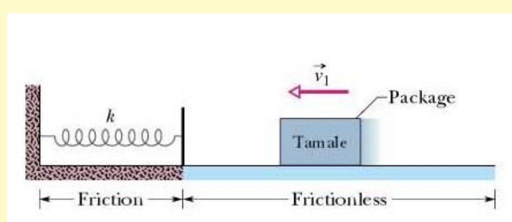
Power

Because work is the transfer of energy, we can rewrite the equations for power using the energy transferred

$$P_{avg} = \frac{\Delta E}{\Delta t} \qquad P = \frac{dE}{dt}$$

Sample 18

A 2.0 kg package of tamales slides along a floor with speed v_1 = 4.0 m/s. It then runs into and compresses a spring, until the package momentarily stops. Its path to the initially relaxed spring is frictionless, but as it compresses the spring, a kinetic frictional force from the floor, of magnitude 15 N, acts on the package. If k = 10,000 N/m, by what distance d is the spring compressed when the package stops?



Sample 19

A circus beagle of mass m = 6.0 kg runs onto the left end of a curved ramp with speed v_0 = 7.8 m/s a height y_0 = 8.5 m above the floor. It then slides to the right an comes to a momentary stop when it reaches a height y = 11.1 m above the floor. The ramp is not frictionless. What is the increase ΔE_{th} in the thermal energy of the beagle and ramp because of the sliding?