AP Physics C: Mechanics

Greetings,

For those of you who are taking Calculus AB concurrently with AP Physics, I have developed a brief introduction to Calculus that gives you an operational knowledge of the material that you will require to better understand Physics in the way that we approach it on this level. After working through this packet, you should be able to do the operations but not necessarily fully understand them from first principles. Full understanding will come later as you more carefully and more deeply explore the topics with your math teacher. I'm asking you to diligently work through the packet so that you are able to take some simple derivatives and integrals, and complete the practice problems at the end of the packet.

For those of you who have already completed Calculus AB, you should still complete the practice problems at the end of the packet as a refresher towards the end of the summer break.

I will not be spending class time reviewing this material, but I will have an after school session during the first week of school to help firm up your understanding and answer any of your questions. After this session you are responsible for the calculus skills described within the packet. I assume that anyone who chooses to take AP Physics is a serious student, one who will do what is necessary to better her or his chances of success.

Sincerely,

Ms. Wolfe

The Derivative

A) A graphical interpretation

1) Consider the following function $f(x) = x^2 + 2x - 5$. We can write this in variable form as, $y = x^2 + 2x - 5$. Let's look at the graph of this function by plotting a few points. See ***Figure 1.***

x	y
0	-5
1	-2
2	3
-1	-6
-2	-5

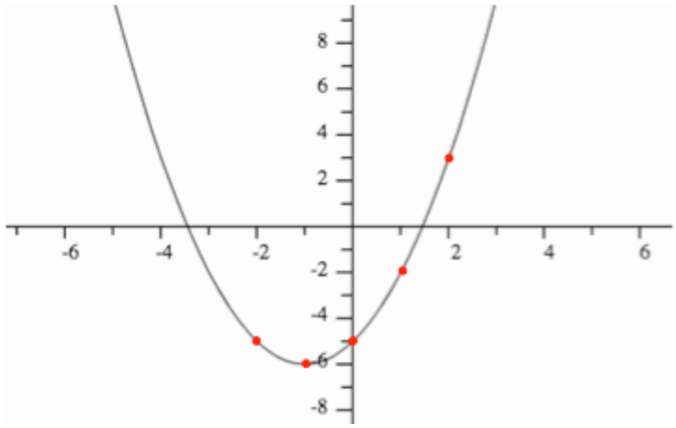


Figure 1

Presto, we have a parabola. We get much useful information from the graph of a function; what its approximate y and/or x-intercepts are, at what point the function is minimum, its line(s) of symmetry, etc. Graphing is a strong yet inexact analytical tool that enables us to get a look at the "big picture" and view the **overall behavior** of a function. Differential calculus is a tool that enables us to exactly analyze its behavior by determining the relative changes in the functional variables **at a point**; namely, ***how the dependent variable (y) increases or decreases for a given change in the independent variable (x)***. Read that last phrase again for understanding. It is a conceptual definition of the derivative of a function. In the end, you will find that being able to determine the behavior of a function at a point is even a more powerful tool than graphing for determining the overall behavior of a function.

2) To understand the statement, ***how the dependent variable (y) increases or decreases for a given change in the independent variable (x)*** let's graphically examine the function more closely between two arbitrary points; say from the point (2,3) to the point (3,10). See ***Figure 2.***

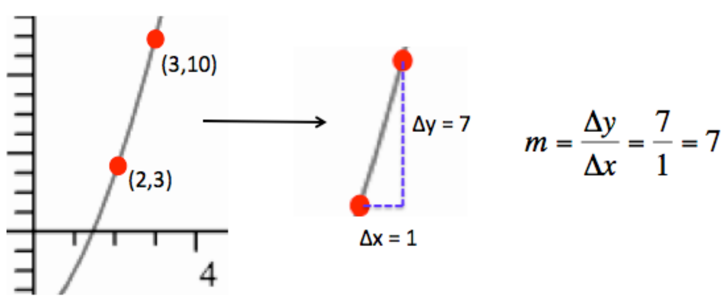
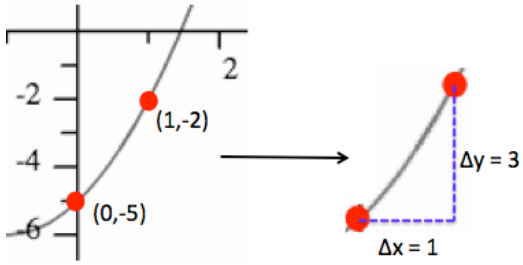


Figure 2

Even though the curve "bends" between these two points we can approximate how y changes with a change in x simply by calculating the slope of a "secant" line passing through both points. The size of this slope, 7 units, gives us an idea of how the function behaves in this **interval**. Compare this slope with that for the two points (0,-5) and (1,-2). See ***Figure 3.***



$$m = \frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$$

Figure 3

Obviously the function increases more rapidly with x between (2,3) and (3,10), where the slope of secant line = 7, than for the interval between (0,-5) and (1,-2), where the slope of the secant line = 3. Of course we can sort of "see" this just by looking at the plot in these intervals, but that's not true of all functions (This is a simple parabola and its graph is well known. Not all functions have such a simple graphical representation.). The calculations of the slope of the secant through the two chosen points **quantitatively reinforces** what we "see" graphically. In summary, this method of finding slopes of secant lines tells us something about how a function behaves in a particular interval, and we can use this method to determine its behavior over any chosen interval for which the function is defined. Even though this process is quantitative, it is not telling us the behavior of the function exactly point by point.

3) Now instead of an interval, suppose we would like to determine the behavior of a function at a **point**. This would mean taking smaller and smaller differences between starting and ending x values when we calculate the slope of the secant lines. For example, using the same function as above, let's start at points (2,3) and (3,10) (a difference in x values of 1 unit), and consecutively divide this x-difference in half, and calculate the slopes of the resulting secant lines. ***Table 1*** givers the results over several such decreasing intervals for the function $y = x^2 + 2x - 5$. You should verify a few of these calculations to make sure you understand the process.

Start point	End point	Δx	Δy	slope = $\Delta y/\Delta x$
(2,3)	(3,10)	1	7.00	7
(2,3)	(2.5,6.250)	0.500	3.25	6.5
(2,3)	(2.25,4.560)	0.250	1.56	6.24
(2,3)	(2.125,3.766)	0.125	0.766	6.21
(2,3)	(2.0625,3.379)	0.0625	0.3789	6.063
(2,3)	(2.03125,3.189)	0.03125	0.1885	6.031

Table 1

The slope ***appears*** to be getting close to the value of 6 as the x value of the end point gets closer and closer to 2. This means we may **suggest** that the slope of the **tangent** line **at** (2,3) is 6. This would be a measure of how the function is changing at the point (2,3). ***Figure 4*** shows sketches that help illustrate this process.

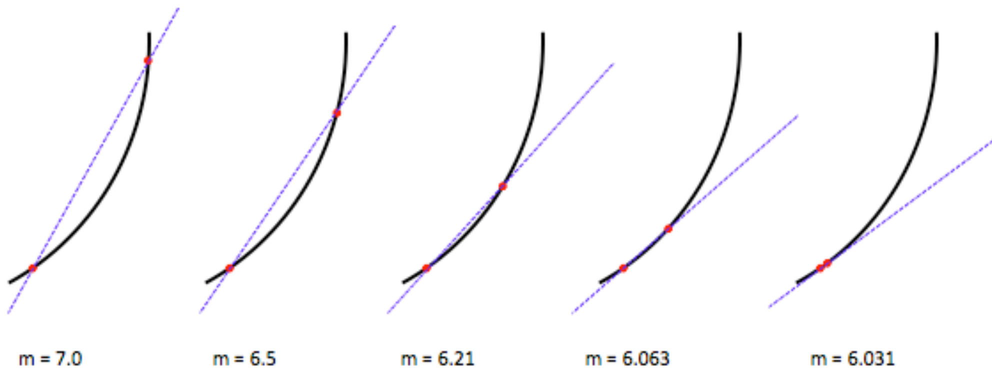


Figure 4

This process is a slow and laborious. Worse yet, we have to use the words ***appears*** and **suggest** rather than the word ***is***, when referring to our result. There must be a better way! Indeed there is. There is a quantitative, analytic method of determining the slope of the tangent lines. It's called the **derivative** of the function. The paragraph below summarizes this section to help keep things in perspective.

A **function** gives the prescription of how the dependent variable, y, depends on the independent variable, x. In the sample given, the prescription is to square the x term add it to twice the x term and than add it to the opposite of five, $y = x^2 + 2x - 5$. The **derivative** of a function is a relation, which indicates how the change in the independent variable effects the change in the dependent variable of the function. Geometrically this is the collection of all slopes of the tangents to the graph of the function for all values for which it is defined. This collection of slope value gives us more information about the behavior of a function at any point. This is illustrated below in ***Figure 5***.

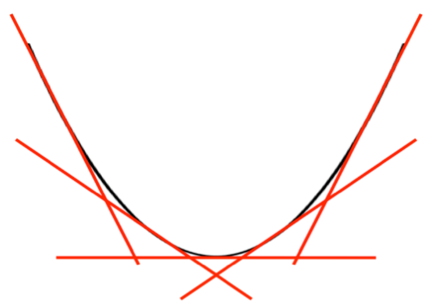


Figure 5

B) An analytical interpretation of the derivative

1) To really find out how a function is changing at a point we must leave the strictly visual realm and enter the purely mathematical realm. Instead of trying to use graphical entities we will generalize the situation to arbitrary

mathematical points. Therefore, instead of particular points such as (2,3) and (2.5,6.25), we will use (x,f(x)) and ($x + \Delta x$,f($x + \Delta x$)), where Δx represents an arbitrary increment of the independent variable x. This means that the generalized form of the slope of the secant line would be,

$slope = \dfrac{\left[\left(f(x+\Delta x\right) - f(x)\right]}{(x+\Delta x) - x}$, which easily reduces to $slope = \dfrac{\Delta x}{\Delta y} = \dfrac{\left[\left(f(x+\Delta x\right) - f(x)\right]}{\Delta x}$ (***eq. 1***). Now for the particular case where $x = 2$, f(x) = $x^2 + 2x - 5$ and $\Delta x = 0.5$, $\dfrac{6.25-3}{2.5-2} = \dfrac{3.25}{0.5} = 6.5$ **as before** (**Table 1**).

Now, in mathematics, after we establish the particular case, we generalize the procedure for **any** case. To find the slope of the tangent to the curve at any point for the given function, we allow Δx to become infinitesimally close to zero. **Graphically**, as mentioned above, that means the secant line becomes a line that intersects the plot at a single point; hence, the **limit process** gives the **slope of a tangent line** at any point (x, f(x)). **Analytically** it tells us how the function changes **at a point** rather than **over an interval**. The process is called a limit and is denoted by the notation $\lim\limits_{\Delta x \to 0}$. The limit of expression (***eq. 1***) as $\Delta x \to 0$ is called the derivative, $\dfrac{df(x)}{dx}$, of the function f(x) (with respect to the variable x), and $\dfrac{df(x)}{dx} = \lim\limits_{\Delta x \to 0} = \dfrac{\left[\left(f(x+\Delta x\right) - f(x)\right]}{\Delta x}$ (***eq. 2***). This limit tells us **quantitatively** how the function is varying at a **point**. Geometrically, it is the **slope of the tangent to the curve at that point**. Clear? If not, read it again until it is clear. It's important.

(***Note*** $\dfrac{df(x)}{dx}$***, f '(x),*** $\dfrac{dy}{dx}$***, and y' are equivalent notations for the derivative of a function*****:**.)

2) Now let us go back to the graphical representation of the derivative for a moment. The process that we used in section (**1A**) determined slopes of lines that passed through **two points** on the graph of a function i.e. a slope of a secant line. Then we chose the points such that they were closer and closer together, to give us information about the value of the **slope of a tangent line** through a particular point of the plot. This gave us information about how the function was varying at that point. But, we had to use qualifying words like **appears** and **suggests** when referring to our results. Now we will use the derivative process to get **a number** that represents the **slope of a tangent line**. This number tells us how the function is varies at that point without having to use words like **appears** and **suggests**. Let's use the limit process describe in section (**1B**) to find the derivative of the function f(x) = $x^2 + 2x - 5$. The calculation is as follows:

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\left\{\left[(x+\Delta x)^2 + 2(x+\Delta x) - 5\right] - \left[x^2 + 2x - 5\right]\right\}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\left(\cancel{x^2} + 2x\Delta x + \Delta x^2 + \cancel{2x} + 2\Delta x - \cancel{5} - \cancel{x^2} - \cancel{2x} + \cancel{5}\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta \cancel{x} + \Delta x^2 + 2\Delta \cancel{x}}{\Delta \cancel{x}} = \lim_{\Delta x \to 0} \left(2x + 2 + \Delta x\right) = 2x + 2$$

Notice that the analytical definition of **the derivative** is a **new function** itself, f '(x) = 2x +2. This function represents the set of **all** slopes of **all** the tangent lines to the curve over which the original function, f(x) = $x^2 + 2x - 5$ is defined. If we wish to know the slope of the tangent line through the point (2,3), for example, we simply evaluate f '(x) = 2x + 2 for x = 2, or f '(2) = 2 · 2 + 2 = 6. This is just as we surmised in section (**A**). We could easily ask further questions like: ***What is the slope of the tangent line through point* (1,-2)? *Is there a point where the tangent line has a slope equal to zero*? *If so, what is this point*?** To answer these questions consider that $\frac{df(x)}{dx}$ at (1,-2) is calculated by evaluating f '(x) = 2x + x for x = 1, or f '(1) = 2 · 1 + 2 = 2 +2 = 4.

To find at what point the slope of the tangent is zero, set $\frac{df(x)}{dx}$ equal to 0. Hence, f '(x) = 2x + 2 = 0, implies 2x = -2, and x = -1. Since f(-1) = $(-1)^2 + 2(-1) - 5 = 1 - 2 - 5 = 6$, the tangent line of f($x$) = $x^2 + 2x - 5$ at (-1,6) has a zero slope (it's horizontal). For this point the change in f(x) for an infinitesimally small change in x is zero. This means the function is either a maximum (downward hump) or a minimum (upward hump). We have plotted this function and know that it is a minimum at (-1, 6), but if we hadn't plotted it or if we had a more difficult function the point(s) where the derivative is zero would be a max or min. (You will learn how to tell definitively which it is, max or min, in your Calculus course.)

3) We could use the analytical definition of a derivative to calculate the derivative of any function, but that would be very cumbersome. To simplify the process, we can develop rules for taking derivatives for **classes of functions**. I will give you an abbreviated proof for one of these rules for the class of power functions f(x) = x^n. I will also state without proof several other rules that will enable you to calculate just about any derivative you will encounter in this course. I'll leave the details and the rigor of proof to you and your Calculus teacher.

A proof for the derivative of f(x) = x^n.

From the definition of the derivative $\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^n - x^n\right]}{\Delta x}$

using the binomial expansion $(a + b)^n = a + na^n b + (n - 1)a^{n-1} b^2 + \cdot b^n$ we have

$(x + \Delta x)^n = x^n + nx^n \Delta x + (n - 1)x^{n-1} \Delta x^2 + \cdot \Delta x^n$ substituting into the expression for $\frac{df(x)}{dx}$,

we have $\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^n - x^n\right]}{\Delta x} = \lim_{\Delta x \to 0} = \frac{\left(\cancel{x^n} + nx^n\Delta x + (n - 1)x^{n-1}\Delta x^2 + \cdots \Delta x^n - \cancel{x^n}\right)}{\Delta x}$,

dividing through by Δx leaves only the term nx^{n-1}, since all the other terms are multiplied by some power of Δx which becomes infinitesimally small in the limit. $\therefore \frac{d(x^n)}{dx} = nx^{n-1}$ (***eq. 3***)

Some other useful rules (without proof but from the basic definition of the derivative (***eq. 2***). Note: C denotes a constant.

i) $\frac{dC}{dx} = 0$ (***eq. 4***)

ii) $\frac{d(\mathrm{Cf(x)})}{dx} = \mathrm{C}\frac{df(x)}{dx}$ (***eq. 5***)

iii) ***The Sum Rule*** $\frac{d(f(x)+g(x))}{dx} = \frac{df(x)}{dx} + \frac{dg(x)}{dx} = f'(x) + g'(x)$ (***eq. 6***) (example: $d(x^2 + x)/dx = 2x + 1$)

iii) ***The Product Rule*** $\frac{d(f(x)\times g(x))}{dx} = \frac{df(x)}{dx}g(x) + f(x)\frac{dg(x)}{dx} = f'(x)g(x) + f(x)g'(x)$ (***eq. 7***)

example: $\frac{d((x+1)(x^2))}{dx} = 1\cdot x^2 + (x+1)2x$

iv) ***The Quotient Rule*** $\frac{d(f(x)\div g(x))}{dx} = \frac{\frac{df(x)}{dx}g(x) - f(x)\frac{dg(x)}{dx}}{(g(x))^2} = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$ (***eq. 8***)

example: $\frac{d((x+1)\div(x^2))}{dx} = \frac{1\cdot x^2 - (x+1)2x}{x^4} = -\frac{x^2+2x}{x^4}$

v) ***Rules for sine and cosine functions*** $\frac{d(\sin(x)}{dx} = \cos(x)$ (***eq. 9***) *and* $\frac{d(\cos(x))}{dx} = -\sin(x)$ (***eq. 10***)

vi) ***The Chain Rule*** if $u = g(x)$, then $\frac{df(g(x))}{dx} = \frac{df(u)}{dx} = \frac{df(u)}{du}\cdot\frac{du}{dx}$ (***eq. 11***)

The chain rule seems difficult but in practice it's not so difficult. For example, suppose $f(x) = (x^2 + 1)^3$. Here $u = g(x) = x^2 + 1$, and, therefore, from the ***power rule*** $du/dx = 2x$. Further $f(u) = u^3$, and again from the ***power rule***, $f'(u) = 3u^2$. Putting these two results together yields

$\frac{d(x^2+1)^3}{dx} = 3(x^2+1)^2(2x) = 6x(x^2+1)^2$. The chain rule often gives fledgling Calculus students fits.

Practice is the remedy for such fits.

C) The physical interpretation of the derivative for 1-D motion

1) A derivative is a mathematical entity that has physical significance in many areas of physics. The one we will consider here is motion. Recall that the definition of velocity is the rate of change of the position of an object. The word "rate" indicates that velocity is a quantity that is evaluated in relation to time. Conceptually, it is a measure of instantaneously how much the position of an object changes with time. Mathematically that fits the definition of the derivative. If we are given a function that gives the dependence of a particle's position on time (say, for example $x = 3t^2 - 2$, where x is in meters), we could find its velocity by calculating the derivative of $x = 3t^2 - 2$ with respect to (wrt) t (time). This yields $v = dx/dt = 6t$. We can find the velocity of the particle at

any time t by evaluating the derivative at that time. The velocity at time $t = 0$ is $6 \cdot 0 = 0$m/s, at time t = 3 s it's 6x3 = 18 m/s, etc. Using calculus 1-D velocity has the **simple** and **elegant** mathematical definition, dx/dt. For a free falling object near the surface of the earth the position of an object is given by $y = v_0 t - 1/2gt^2$, where v_0 represents the initial velocity of the particle, and g is the acceleration due to gravity at the earth's surface (9.8 m/s^2). Note, the variable, y, is used to indicate **vertical** position. The vertical velocity of the particle for any time t is $v_y = dy/dt = v_0 - gt$. If $v_0 = 12.5$ m/s, this becomes $v_y = 12.5\text{m/s} - 9.8\text{m/s}^2 t$. Evaluating this expression for any time, t, determines the particle's velocity at that time. So, for example that velocity at time 1.0s is $v_y = 12.5 - 9.8 \cdot 1.0 = 12.5 - 9.8 = 2.7$m/s, at 2.0s $v_y = 12.5 - 9.8 \cdot 2.0 = 12.5 - 19.6 = -7.1$m/s. Given the position of a particle as a function of time, the derivative gives you its velocity.

2) Recall further that the acceleration of a particle is the rate of change of its velocity. That would mean that mathematically acceleration is the derivative of the velocity function, or, $a = dv/dt$. For the example given above $v_y = 12.5 - 9.8t$. Taking the derivative yields $a_y = -9.8\text{m/s}^2$. For the general free-fall position function, $y = v_0 t - 1/2gt^2$, taking the derivative gives $v_y = v_0 - gt$ and, taking the derivative a second time yields $a_y = -g$. In two operations we have shown, in general, that the velocity of a free falling object has a linear dependence on time and its acceleration is constant. Wow, is that **simple** and **elegant**! Simply said, given the velocity of a particle as a function of time, the derivative gives you its acceleration. Furthermore, given the position of a particle as a function of time, the first derivative gives you its velocity, its second derivative gives you its acceleration. (Notation for repeated derivatives is as follows: first derivative $\frac{dx}{dt}$, $\frac{df(t)}{dt}$, f'(t), or x'; second derivative, $\frac{d^2x}{dt^2}$, $\frac{d^2f(t)}{dt^2}$, f''(t), x''; third derivative $\frac{d^3x}{dt^3}$, $\frac{d^3f(t)}{dt^3}$, f '''(t), or x'''; and so on.)

The Integral

A) The definite integral

In doing physics it is often necessary to determine the area-under-the-curve. For example, if a force acts on an object as it moves through some displacement, the work done on the object by the force, is the area under the position-force curve. Suppose you have the function $y = ½\, x + 1$. ***Figure 6a*** shows a sketch of this function.

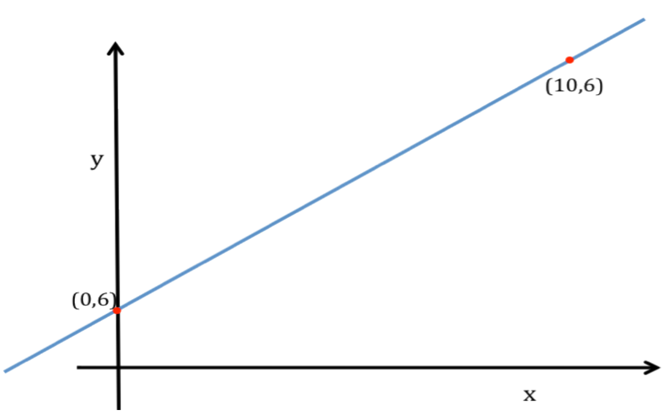


Figure 6a

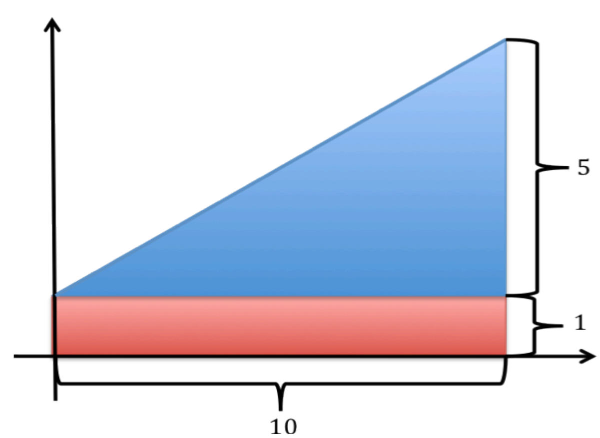


Figure 6b

For this "curve" it is pretty easy to determine the area-under-the-curve between, say, $x = 0$ and $x = 10$. All we need to do is add the areas of two simple geometric figures, a rectangle and a triangle as shown in ***Figure 6b***. As is easily seen, area = ½(10)(5) + 1(10) = 25 + 10 = 35. This is all very easy for straight line "curves". What can we do to calculate the area under a curve like $y = f(x) = \frac{1}{2}x^2 + 1$, a sketch of which is shown in ***Figure 7***?

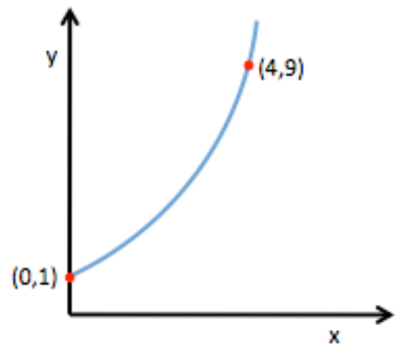


Figure 7

We could approximate the area-under-the-curve for this function by summing several rectangular regions as shown in ***Figure 8a***. There would be some error in this calculation as each of the rectangles has an area that is a little **less than** the actual area. We could **average** the results with the sum taken as in ***Figure 8b*** because these rectangles **over-evaluate** the desired sum. We could take the sum for rectangles with their heights taken at the midpoint as in ***Figure. 8c***. The fact remains, no matter what the method, our results must be modified with words like "**appears"** and "**suggests"**. There's a better way that again involves a limiting process - the **definite integral**.

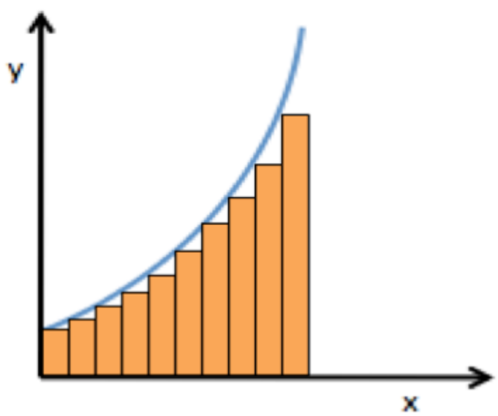


Figure 8a

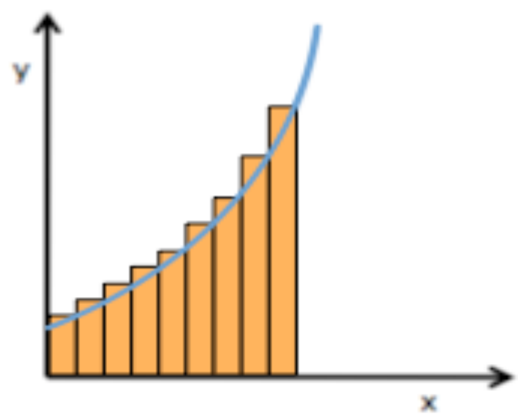


Figure 8b

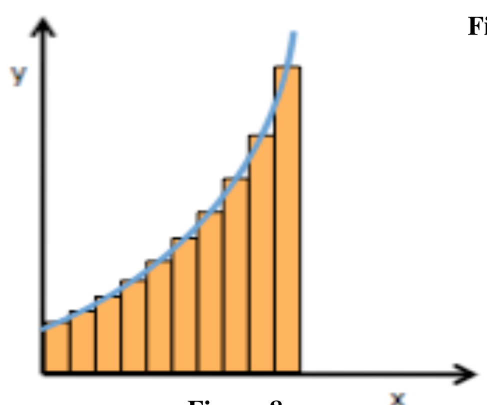


Figure 8c

We start by generalizing to some function f(x) and write an expression for the sum of incremental rectangular areas with heights taken at the midpoint (***Figure 9***).

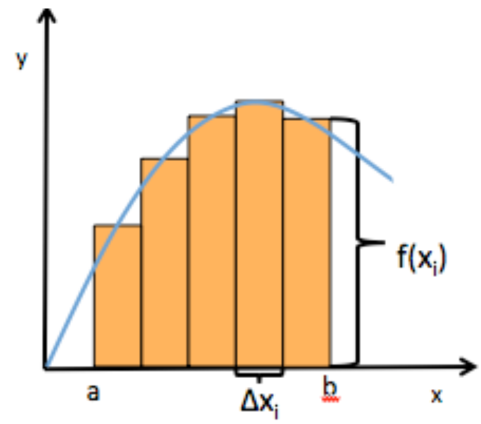


Figure 9

This sum is $f(x_1)\cdot\Delta x_1 + f(x_2)\cdot\Delta x_2 + \cdot\cdot\cdot\ f(x_n)\cdot\Delta x_n = \sum_{i=1}^{n} f(x_i)\Delta x_i$ where $f(x_i)$ represents the midpoint height (the value of the function taken at the midpoint of each rectangle). If we take smaller and smaller widths Δx_i the number of rectangles become infinite in number and infinitesimally narrow ($\Delta x_i \rightarrow dx$, where the differential, dx, represents an infinitesimally small change in x); hence, sum neither over-shoots or under-shoots but is exact. This sum represents the area between the x-axis and the function for the interval it's taken, and is called the **definite integral** for the function over that interval. The definite integral is denoted as follows:

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\cdot\ \Delta x_i = \int_a^b f(x)dx \quad (\textbf{\textit{eq. 12}})$$ a is the beginning value of x and b is the ending value of x.

The definite integral for an arbitrary function is illustrated graphically in ***Figure 10***. (Note: Area above the x-axis is taken as positive and area below the x-axis it is taken as negative.) For this result we do not have to use the words **appears** or **suggests** in reference to this result. It is the **exact area-under-the-curve**.

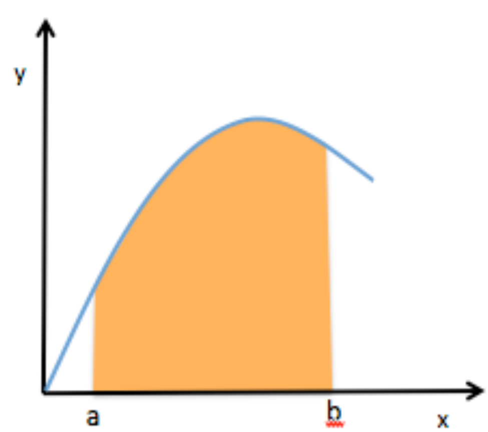


Figure 10

B) The indefinite integral or the anti-derivative

An indefinite integral or anti-derivative is similar to the definite integral but has no particular numerical value. The anti-derivative g(x) of a function f(x) is defined as the function whose derivative is the function f(x). Suppose f(x) = x^2, than the anti-derivative, g(x), would be given by $g(x) = \frac{x^3}{3} + \mathrm{C}$, where C is an arbitrary constant. To verify that this is the **anti-derivative** take the derivative of g(x) see if you get f(x) back. Using the rules for taking derivatives, we get $\int \frac{d(\frac{x^3}{3})}{dx} = \frac{3x^2}{3} = x^2$ *and* $\frac{d(C)}{dx} = 0$ putting these results together gives g '(x) = x^2 = f(x).

Why is there the arbitrary constant, C? Well because $\frac{d(C)}{dx} = 0$, no matter what the value of C. g(x) could be $\frac{x^3}{3} + 1$, or $\frac{x^3}{3} + 1000000$, or even $\frac{x^3}{3} + 0$ for that matter. Any of these work as an anti-derivative. The **indefinite integral** is denoted as $g(x) = \int f(x)dx$. The integral sign for the indefinite integral, $\int$, has no upper or lower bounds. This sign **does not represent** a definite **area** value, it **represents** the **anti-derivative** of the function **f(*x*)**. Consider our initial example y = f(x) = ½ x +1. What is the anti-derivative for this function? We have to think backwards to get this result. We ask ourselves, what function has a derivative equal to ½ x? Thinking backwards we arrive at the answer, ¼ x^2. Then we ask what function has a derivative equal to 1? The answer is, x. Adding the arbitrary constant gives g(x) = ¼ x^2 + x + C. We then convince ourselves that our result is the anti-derivative by taking the derivative of $x^2/4 + x$ + C, for which we get $2 \cdot x/4$ +1 + 0 = ½ x+ 1. We get y = 1/2x + 1 back! This shows that ¼ x^2 + x + C is the anti-derivative of y = ½ x + 1. In a short, we say that the integral of f(x) = ½ x + 1 is ¼ x^2 + x + C, in symbol form this is $\int f(x)dx = \int (\frac{1}{2}x^2 + 1)dx = \frac{1}{4}x^2 + x + \mathrm{C}$ (Note: when we say integral we mean indefinite integral or anti-derivative). When we talk about a definite integral, we say "definite integral" with its bounds.

C) The relationship between the definite integral and the indefinite integral

The **definite integral** of a function is simply its **indefinite integral** evaluated over the interval for which you would like to determine the area-under-the-curve. The definite integral has physical significance the indefinite integral does not. (for example if F(x) represents a force then $\int_a^b \mathrm{F}(x)dx$ represents the work done by that force over the interval a to b.) You would determine the area-under-the- curve in any interval just by evaluating the derivative over that interval. For f(x) = ½ x + 1 in the interval between x = 0 to x = 3, you would have,

$$\int_0^3 (\tfrac{1}{2}x + 1)dx = \tfrac{1}{4}x^2 + x\Big|_0^3 = (\tfrac{1}{4}3^2 + 3) - (\tfrac{1}{4}0 + 0) = \tfrac{9}{4} + 3 = 5.25$$

This number represents the area-under-the-curve for y = ½ x + 1 between x = 0 and x = *3*.

The prescription for calculating a definite integral is:

1) Evaluate the anti-derivative at the upper bound.

2) Evaluate the anti-derivative at the lower bound and subtract it from the value of the upper bound result.

For instance, using the same function, $y = ½\, x + 1$ the area for the bounds $x = 2$ to $x = 8$ would be,

$$(8^2/4 + 8) – (2^2/4 + 2) = 24 – 3 = 21.$$

This is a method for calculating definite integrals that is given without proof or full explanation. To understand integral calculus you first must have a thorough understanding of differential calculus and certainly a more rigorous treatment of the subject. **What I expect at the start is the ability to perform the calculation of some simple definite and indefinite integrals like those given as examples. I am not expecting depth of understanding.**

Practice Problems

Please, do not write on this paper. Do all work on a separate sheet of paper and show all work.

1) Find the slope of the tangent at the point given

a) $y = f(x) = 2x^2 + 7$ at (-3,25)

b) $y = f(x) = 3x^4 - x^3 + 3$ at (0,-1)

2) The position in meters of a particle is given by $x = t^3 - 2t^2 + 6$

a) Where is the particle at time t = 0 and t = 2s?

b) What is its speed at t = 0 and t = 2s?

c) What is its acceleration at t = 0 and t = 2s?

3) Evaluate the following definite integral

$$\int_1^3 (t^2 + t + 1)dt$$

4) The velocity of a particle is given by $v(t) = 3t^2 - 4t - 5$;

a) Integrate to find the position as a function of time.

b) Evaluate the integral between 0 and 2s to determine the displacement of the particle between these times.

5) The acceleration of a particle is given by $v(t) = -t + 3$

a) Integrate to find the velocity as a function of time.

b) Evaluate the integral between 1 and 3s to determine how much the velocity of the particle changes in this time.